

Spin–orbit and tensor interactions in homogeneous matter of nucleons: accuracy of modern many–body theories

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Abstract

We study the energy per particle of symmetric nuclear matter and pure neutron matter using realistic nucleon–nucleon potentials having non central tensor and spin–orbit components, up to three times the empirical nuclear matter saturation density, $\rho_0 = 0.16 \text{ fm}^{-3}$. The calculations are carried out within the frameworks of the Brueckner–Bethe–Goldstone (BBG) and Correlated Basis Functions (CBF) formalisms, in order to ascertain the accuracy of the methods. The two hole–line approximation, with the continuous choice for the single particle auxiliary potential, is adopted for the BBG approach, whereas the variational Fermi Hypernetted Chain/Single Operator Chain theory, corrected at the second order perturbative expansion level, is used in the CBF one. The energies are then compared with the available Quantum and Variational Monte Carlo results in neutron matter and with the BBG, up to the three hole–line diagrams. For neutron matter and potentials without spin–orbit components all methods, but perturbative CBF, are in reasonable agreement up to $\rho \sim 3 \rho_0$. After the inclusion of the LS interactions, we still find agreement around ρ_0 , whereas it is spoiled at larger densities. The spin–orbit potential lowers the energy of neutron matter at ρ_0 by $\sim 3\text{--}4$ MeV per nucleon. In symmetric nuclear matter, the BBG and the variational results are in agreement up to $\sim 1.5 \rho_0$. Beyond this density, and in contrast with neutron matter, we find good agreement only for the potential having spin–orbit components.

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Homogeneous matter of nucleons plays an important role in modern nuclear physics. For instance, short range correlations in nuclei, induced by the strong nucleon–nucleon (NN) interaction, are expected not to be very different from those in infinite nuclear matter at a corresponding local density. This prediction has found quantitative confirmation in the studies of quantities like inclusive and exclusive electron scattering cross sections. In general, the study of infinite matter of nucleons, starting from a hamiltonian containing realistic NN interactions, shows dramatic departures from the predictions of the independent particle models (IPM)[1]. A clear example is the depletion of the momentum distribution, $n(k)$, at momenta below the Fermi momentum, $k < k_F$, and the corresponding appearance of a large momentum tail [2, 3, 4, 5, 6, 7], otherwise absent in any IPM. An object of intensive study in this field is the equation of state (EOS) of nuclear matter, both for symmetric nuclear matter (SNM) and pure neutron matter (PNM). The aims are (i) testing the validity of microscopic interactions, fitted to the properties of the light ($A=2$ and 3) nuclei, in a many–body system and (ii) checking the accuracy of the adopted methodologies in a demanding environment. Moreover, an accurate knowledge of the EOS, and in particular of the density dependence of the symmetry energy [8], is needed in order to determine, with the highest possible level of confidancy, the structure and the thermal evolution of the neutron stars [9]. In this respect, it is compulsory to use both realistic hamiltonians as well as reliable many–body techniques.

From the point of view of the NN interaction, large progresses have been achieved in the last decade. Modern potentials [10, 11, 12] are “phase shift equivalent”, since all fit a huge set of NN scattering data [13] below 350 MeV with χ^2 per datum close to 1. Three-nucleon forces have been also introduced, and the resulting hamiltonian provides a nice reproduction of the binding and low–lying states energies of light nuclei ($A \leq 10$) [14, 15].

In parallel to the advances in the knowledge of the nuclear interaction, and partly motivated by its strong state dependence, several many–body theories have been noticeably pushed forward. In this paper we use, and compare, the Correlated Basis Functions (CBF) [16, 17] and the Bethe–Brueckner–Goldstone (BBG) [18] theories.

CBF is particularly suited to deal with strongly interacting systems, since the non–perturbative correlation effects induced by the interaction are directly embedded into the basis states through an appropriate many–body correlation operator acting on some given model wave function. It is clear that complicated interactions are expected to induce similarly non trivial correlations. For instance, (i) the strong one–pion–exchange potential,

essential to provide the nuclear binding, reflects into the existence of a long-medium range tensor-like correlation, whereas (ii) the strong, short range (about 0.5 fm) NN repulsion, preventing the nuclear systems from collapsing, generates a mostly central correlation so strong that the wave function is almost vanishing at these low internucleon distances. Operator matrix elements and expectation values between CBF states are, by far, more realistic than those evaluated in a free Fermi Gas (FG) basis. As a consequence, a CBF based perturbative expansion is expected to converge much faster. The obvious drawback is that the matrix elements are difficult to be accurately computed. The zeroth order of the CBF perturbative theory corresponds to the purely variational estimates, since the correlated ground state wave functions are fixed by minimizing the energy with respect to the correlation parameters. Within CBF the matrix elements can be computed either by cluster expansions in Mayer-like diagrams and integral equations methods or by Monte Carlo (MC) based evaluations. The Fermi Hypernetted Chain/Single Operator Chain (FHNC/SOC) [19] equations belong to the first type of approach and their solution provides the sum of infinite classes of cluster diagrams. However, the FHNC-like summation is not complete and some diagrams, like the “elementary” ones, are not fully considered. This fact constitutes an approximation within the theory, that must be checked against known exact results, as sum rules, or compared with the outcomes of other methods. Variational Monte Carlo (VMC) [14] provides an alternative and exact, but expensive, way of completely sum the cluster contributions through stochastic evaluation of the needed many-body integrals. The integral equation method has the advantage of not being limited by the number of particles, whereas VMC becomes impractical at large A -values.

Standard perturbation theory cannot be straightly applied to the nuclear case because of the strong, non perturbative, repulsive core. In the BBG theory the *in medium* two-body scattering G -matrix is introduced. The G -matrix has a regular behavior even for strong short-range repulsions, and it constitutes the starting point to derive the so-called *hole-line expansion*, where the perturbative diagrams are grouped according to the number of independent hole-lines (see *e.g.* Ref.[18]). The Brueckner-Hartree-Fock (BHF) approximation sums only two hole-line diagrams (2HL), and it is expected to take accurately care of the two-body correlations. Within BHF, one has also to self-consistently consider the auxiliary single particle potential, $U(k)$, and the diagrams having *potential insertions*. It has been recently shown that in the *continuous choice* for the single particle potential (see Eq.(5)),

the three hole-line (3HL) correction is small up to densities several times larger than ρ_0 [20].

Quantum Monte Carlo (QMC), in its different implementations, as Auxiliary Field Diffusion MC (AFDMC) [21] and Green's Function MC (GFMC) [14], allows in principle for an *exact* solution of the many-body Schrödinger equation. However, its accuracy is limited by the *fermion sign problem* [22], solved in the aforementioned methods by limiting the sample walk inside a fixed nodal surface. This *fixed node* approximation, which is essential to get reliable numerical estimates in systems with a large number of fermions, makes the present day QMC results essentially variational. The more accurate are the nodes, the closer to the true eigenvalues are the QMC outcomes. AFDMC and GFMC differ in the way they treat the spin degrees of freedom. The first method samples the spin states by introducing auxiliary fields of the Hubbard-Stratonovich type, whereas the second one sums them completely. The advantage of AFDMC lies in the fact that simulations with a rather large number of particles (up to $A \sim 60$ for PNM [23]) can be carried on with a low variance. In contrast, GFMC has been limited so far to $A=14$ [24] in PNM, with the obvious consequence of showing large finite size corrections. None of these QMC methodologies has been so far applied to SNM.

The nuclear interaction has important tensor and momentum dependent (MD) components. The tensor potential is essentially due to One Pion Exchange and is the main responsible for the nuclear binding. For the Argonne v_{18} [10] (A18) the NN tensor interaction is dominant in the deuteron ($S = 1, T = 0$) ground state, contributing to the energy per nucleon as $\langle v_t \rangle = -8.34$ MeV, whereas the MD potential gives only $\langle v_{MD} \rangle = -0.52$ MeV. However, in the ^3H nucleus, the contributions are $\langle v_t \rangle_{T=0} = -10.89$ MeV, $\langle v_{MD} \rangle_{T=0} = -1.97$ MeV, and $\langle v_t \rangle_{T=1} = -0.14$ MeV, $\langle v_{MD} \rangle_{T=1} = 1.52$ MeV [25]. We stress that the $T = 1$ isospin triplet channels are the only ones effective in PNM. The evaluation in heavier systems, either nuclei ($A > 12$) or infinite matter, appears methodologically problematic. For neutron matter, the GFMC and AFDMC estimates of the LS contribution employing the Argonne $v_{8'}$ (A8') [26] potential are in disagreement [23, 24]. A8' is a simplified version of A18, truncated after the linear spin-orbit components and refitted to have the same isoscalar part of A18 in all the S and P waves, as well as in the 3D_1 wave and its coupling to the 3S_1 . It has been used in GFMC calculations of light nuclei since the difference with A18 is small and can be treated perturbatively [26].

In this letter we make a comparison between the BHF and CBF energies in neutron and

nuclear matter, paying a particular attention to the tensor and spin–orbit components of the NN interaction. The energies are computed up to three times ρ_0 , considering the second order CBF perturbative corrections to the variational estimates and, when available, the three hole–line contributions to the BHF approximation. The accuracy of the calculations are then assessed, according to the approximations in the cluster and the hole–line expansions. We do not include three–nucleon interactions (TNI), since we are mostly interested in just evaluating the reliability of different many–body techniques in a wide range of densities. Moreover, the level of confidence in the available TNI is not the same as in the NN ones, both for the objective larger difficulties in the corresponding theoretical frameworks and for the much smaller three–nucleon experimental data set, to be fitted by the theoretical TNI.

CBF calculations in homogeneous matter of nucleons are based upon the set of correlated basis functions,

$$\Psi_n^{CBF} = \left(\prod_{i<j} f_{ij} \right)_{sym} \Psi_n^{FG}, \quad (1)$$

built by applying a symmetrized product of two–body correlation operators, f_{ij} , to the Fermi Gas (FG) states, Ψ_n^{FG} . In nuclear systems f_{ij} depends on the internucleon distance, as well as on the relative state of the pair. We adopt a correlation operator having the same state dependence as the A8' potential,

$$f_{ij} = \sum_{p=1,8} f^p(r_{ij}) O_{ij}^p, \quad (2)$$

where $O_{ij}^{p=1,3,5,7} = 1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L} \cdot \mathbf{S}$ and $O_{ij}^{p=2,4,6,8} = O_{ij}^{p-1} \tau_i \cdot \tau_j$, being S_{ij} and $\mathbf{L} \cdot \mathbf{S}$ the tensor and spin–orbit operators.

Variational estimates of the energy are obtained by minimizing the expectation value of the hamiltonian, $H = T + V = -\sum_i \hbar^2 \nabla_i^2 / 2m_i + \sum_{i<j} v_{ij}$, on the correlated ground state, $\Psi_0^{CBF} = (\prod_{i<j} f_{ij})_{sym} \Psi_0^{FG}$, where Ψ_0^{FG} is the usual FG ground state of an infinite matter of fermions, and we consider two–body interactions only. The correlation operator is determined by solving the Euler equations corresponding to the minimization of the energy at the two–body level of the cluster expansion. The free parameters of this procedure are the healing distances, d_c and d_t , of the central and tensor components of f_{ij} and the quenching factor, α , of the spin dependent part of v_{ij} , adopted in the solution of the Euler equations [19].

Within the cluster expansion method, the variational ground state energy,

$$E_0^v = \langle \Psi_0^{CBF} | H | \Psi_0^{CBF} \rangle / \langle \Psi_0^{CBF} | \Psi_0^{CBF} \rangle, \quad (3)$$

is computed by means of the FHNC/SOC theory and its improvements [19, 27]. The FHNC/SOC equations sum diagrams containing an infinite number of nucleons. However, the sum is incomplete since there are two classes of diagrams not considered: (i) *elementary* diagrams, contributing to the energy at least as ρ^3 , since the lowest order contribution comes from four-body diagrams; (ii) corrections, Δ , due to the non complete inclusion of all the correlation components (2) with $p \geq 2$. $\Delta = 0$ for state independent correlations, $f^{p \geq 2} = 0$, whereas it is non zero in the state dependent case. Three-body clusters give the lowest order contribution to Δ , which behaves at least as ρ^2 . Δ^{MI} due to the momentum independent part of the correlation, $f^{p \leq 6}$, has been recently evaluated in SNM and PNM [28]. The results for A18, as extracted from Tables X and XI of the Reference, are: $\Delta^{MI}(\text{SNM}, \rho = \rho_0) = -2.6$ MeV, $\Delta^{MI}(\text{SNM}, \rho = 1.5 \rho_0) = -2.8$ MeV, $\Delta^{MI}(\text{PNM}, \rho = \rho_0) = 0.4$ MeV, and $\Delta^{MI}(\text{PNM}, \rho = 2 \rho_0) = -1.4$ MeV. The percentile contributions to the total three-body cluster energies are: -26% , -18% , 3% and -6% , respectively [28]. The magnitude of Δ^{MI} points to a good accuracy of the FHNC/SOC estimates in the PNM case. In addition, asymmetric nuclear matter, with a low percentage of protons (as of interest in neutron star physics), should be safely addressed by the same technique.

We include in our calculations the lowest order four-body elementary diagrams, $E_{ee}^{(4)}$, linear in the correlations, $[f^1(r)]^2 - 1$ and $2f^1(r)f^{p \geq 2}(r)$. $E_{ee}^{(4)}$ belongs to the *exchange-exchange* elementary diagrams subclass and it is expected to be relevant for potentials having large Majorana components [29]. The other elementary diagrams have a higher power dependence on the correlations and are not considered in this work.

The CBF energy is evaluated by adding the second order CBF perturbative corrections to the variational estimate, $E_0^{CBF} = E_0^v + \Delta E_0^{CBF}$. ΔE_0^{CBF} is computed by considering two-particle two-hole intermediate correlated states, $\Psi_{2p2h}^{CBF} = (\prod_{i < j} f_{ij})_{sym} \Psi_{2p2h}^{FG}$, where Ψ_{2p2h}^{FG} is the FG $2p-2h$ state. Ψ_{2p2h}^{CBF} is then normalized and orthogonalized to Ψ_0^{CBF} . The evaluation of ΔE_0^{CBF} is carried on as explained in Ref. [16], at low (second and part of the third) orders of the cluster expansion of the non diagonal matrix elements of the hamiltonian.

In the BHF approximation the Brueckner G-Matrix, $G(\omega)$, is obtained by solving the

Bethe–Goldstone equation,

$$\begin{aligned} & \langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | v | k_3 k_4 \rangle \\ & + \sum_{k'_3 k'_4} \langle k_1 k_2 | v | k'_3 k'_4 \rangle \frac{Q(k'_3, k'_4)}{\omega - e(k'_3) - e(k'_4)} \langle k'_3 k'_4 | G(\omega) | k_3 k_4 \rangle, \end{aligned} \quad (4)$$

where $Q(k, k')$ is the Pauli operator ($Q(k, k')=1$ if both arguments are larger than k_F and $Q(k, k')=0$ otherwise) enforcing the scattered momenta to lie above the Fermi level. The single particle potential, $U(k)$, entering the definition of $e(k) = \hbar k^2/2m + U(k)$, should be self-consistently determined together with the G-Matrix and it is given by

$$U(k) = \sum_{k' < k_F} \langle k k' | G(e(k) + e(k')) | k k' \rangle_a, \quad (5)$$

where the suscript a indicates antisymmetrization of the matrix elements. This choice for $U(k)$ has been shown to considerably improve the convergence of the hole–line expansion [20]. Finally, the BHF energy is computed via:

$$E_0^{BHF} = \sum_{k_1 < k_F} \frac{\hbar^2 k_1^2}{2m} + \frac{1}{2} \sum_{k_1, k_2 < k_F} \langle k_1 k_2 | G(e(k_1) + e(k_2)) | k_1 k_2 \rangle_a. \quad (6)$$

We solve the G–matrix equation (4) by expanding it in partial waves up to $J = 8$ and adopting the Born approximation for $J = 9 - 20$. The accuracy of this approximation has been checked to be valid within a few percent [30]. The first correction to BHF arises from the three hole–line diagrams. In this respect, a three–body scattering matrix, $T^{(3)}$, is introduced, which satisfies the Bethe–Fadeev integral equation [18, 31]. In PNM the three hole–line contribution is just a few percent of the BHF energy up to several times ρ_0 . Concerning the TNI, it has to be stressed that its correct evaluation in the BBG approach would require their inclusion in the Bethe–Fadeev equation. However, TNI are presently treated within the BHF approximation, averaging over the third nucleon coordinates [32]. In any case, as it was mentioned before, in the present paper we do not consider three-body forces.

We first present in Table I the energies per nucleon for Argonne $v_{6'}$ (A6'), a simplified version of Argonne $v_{8'}$, containing only its same static part, without spin–orbit components and without refitting the phase shifts. The Table gives the FHNC/SOC variational energies, E_0^v , those at the second order of the CBF perturbative expansion, E_0^{CBF} , and the BHF approximation estimates, E_0^{BHF} . For PNM we also show the three hole–line expansion [33],

E_0^{BBG} , the GFMC [24] and the AFDMC [34] energies. The GFMC column gives the energy of 14 neutrons in a periodic box (PB), including the finite size box corrections, coming mostly from the long range part of the interaction (beyond the box boundaries) and from the difference in the kinetic energies between free neutrons in a box and the homogeneous Fermi Gas. The box corrections for A6' were not given in Ref. [24], so we have used those of the A8' model, on the basis that the long range potential correction comes largely from the tensor and spin interactions, not from the spin-orbit ones. The same number of neutrons in a periodic box has been used in the AFDMC estimates, but with a presumably more accurate treatment of the finite size corrections. In PNM the 3HL diagrams included in E_0^{BBG} lower E_0^{BHF} at any density. The comparison with GFMC and AFDMC shows that in PNM the BBG and variational theories both give estimates for the energies accurate within a few percent. The CBF corrections are, by construction, negative and lower E_0^v by ~ 2 MeV at ρ_0 . They rapidly increase in absolute value with density and this fact may be viewed as an indication that the attractive CBF correction might be compensated by other contributions (many-body cluster diagrams not included in the evaluation of the correlated matrix element, correlated $n > 2$ particle-hole intermediate states and/or higher order perturbative contributions). The BHF and variational energies of SNM are satisfactorily close up to ρ_0 , and start to differ at higher densities, E_0^v being increasingly lower than E_0^{BHF} . The CBF corrections are as large in SNM as in PNM and worsen the disagreement. It is likely that the main reason for this behavior lies in the strong tensor interaction. In fact, we can extract from the BHF results $\langle v_t + v_{t\tau} \rangle_{\text{SNM}} = -25.58$ MeV for A6' at ρ_0 . In SNM there are no 3HL/BBG and Quantum Monte Carlo results for A6'. The 3HL contributions in SNM have been computed for A18 [33] and found to be repulsive at low densities, up to $\sim \rho_0$, and mostly attractive above. A quantitative evaluation of these corrections would be of great help to assess more completely the convergence of the hole-line expansion in SNM.

Table II gives the energies per nucleon for the Argonne v_8 interaction. Again we give the 3HL/BBG [33], GFMC [24] and AFDMC [34] energies for PNM. The GFMC column reports in parentheses also the PNM finite size box corrections. These corrections are impressively large and point to the need of GFMC simulations with a larger number of particles in order to ascertain their relevance. We have already stressed that AFDMC seems to be more accurate in treating finite size effects, since the simulation is fully tail corrected [34]. The difference between box corrected GFMC and AFDMC energies at saturation density is small

for the A6' potential, but it is larger in the A8' one. An additional source of difference may be the use of an unconstrained path simulation in GFMC, lowering the PNM A8' energy by ~ 1.5 MeV at ρ_0 , whereas AFDMC uses a constrained path. Once more, the differences are smaller for A6'. The 3HL diagrams in PNM lower E_0^{BBG} with respect to E_0^{BHF} up to $\rho/\rho_0=1.5$, whereas their contribution is repulsive at higher densities, contrary to the A6' model. The CBF corrections behave as in A6', being ~ -2 MeV in PNM at ρ_0 and more and more attractive with increasing density. BBG is in good agreement with the GFMC energies, even if, strangely enough, BHF is in better accordance. The variational and CBF methods appear to work rather well up to ρ_0 , whereas are too attractive beyond it. A complete analysis of the missing cluster terms in presence of spin-orbit terms, both in the potential and in the correlation (as the one performed for the spin and isospin case [28]), has not yet been done. Work in this direction is in progress. AFDMC is rather more repulsive than the other methods. Part of the discrepancy with GFMC has been resolved in Ref. [35] by a better choice of the nodal structure of the guiding function. This improvement has lowered the AFDMC neutron matter energy in the Periodic Box model of A8' by 1 (1.6) MeV at ρ_0 ($2\rho_0$). For SNM the FHNC/SOC and BHF methods give close energies at densities larger than ρ_0 , whereas CBF appears to work better at $\rho \leq \rho_0$. Again we stress the need of evaluating the 3HL/BBG corrections. If they are $\sim 1-2$ MeV, as for A18, then there are indications that for A8' FHNC/SOC is more accurate in SNM than in PNM and that the CBF corrections are overestimated. It results also clear that A8' provides a saturation point for SNM that is exceedingly too large and should be used with some care in studying high-density nuclear systems (see neutron stars).

In Table III the contributions to the energy per nucleon in BHF of the interaction components are given for the A6' and A8' models. The BHF approach provides the total energy, but does not give access to the separate contributions of the potential and kinetic energy in the correlated ground state. However, it has been recently shown that the Hellmann-Feynmann theorem can be used to calculate the separate contribution of the different components of the NN interaction to the potential energy [36]. Then the kinetic energy is evaluated by subtracting the potential energy from E_{BHF} . The spin-orbit potential gives a large contribution, $\langle v_{LS}(\rho = \rho_0) \rangle \sim -9$ MeV and $\langle v_{LS}(\rho = 2\rho_0) \rangle \sim -22$ MeV, for both PNM and SNM. However, $\Delta E_{LS} = \langle H_{A8'} \rangle - \langle H_{A6'} \rangle$ results smaller, especially at ρ_0 , where $\Delta E_{LS}(\rho = \rho_0) \sim -3.9$ MeV in PNM. This result is in line with the other estimates, since

$\Delta E_{LS}(\rho = \rho_0) \sim -3.3$ MeV in BBG, -3.9 MeV in FHNC/SOC, -4.1 MeV in CBF, and -2.7 MeV in GFMC. The AFDMC energies reported here give $\Delta E_{LS}^{AFDMC}(\rho = \rho_0) \sim -1$ MeV. However, as pointed out before, improved estimates of E_0^{AFDMC} [35] could bring this difference down to ~ -1.2 MeV. Similar numbers are obtained for SNM, where $\Delta E_{LS}(\rho = \rho_0) \sim -4.4$ MeV in BHF, -3.1 MeV in FHNC/SOC and -3.8 MeV in CBF.

To conclude this letter, we compare in Table IV the SNM energies per nucleon as obtained with the A8' and A18 potentials. The Table gives the BHF results of this paper, the BHF ones of Ref. [30] (BHF/BF), the BBG of the same paper and the FHNC/SOC. The comparison between BHF and BHF/BF is shown to verify how accurate are the different treatments, given by the two independent calculations, of the approximations required to solve the BHF equation with a large number of partial waves and at high densities. The difference is at most of ~ 5 % at the highest density, $\rho/\rho_0=4.5$, except where the A18 energies are close to zero, at $\rho/\rho_0=3.38$, where they are anyhow small. A18 has a richer operatorial structure than A8', including $(\mathbf{L} \cdot \mathbf{S})^2$ and L^2 terms, as well as isovector and isotensor ones. Cluster diagrams including these extra components have been computed in FHNC/SOC only at some low order level (two-body and a few three-body diagrams). It may be expected that higher order diagrams become increasingly more important with the density, as it is suggested by the raising difference between FHNC/SOC and BHF/BBG at densities above $\sim 2 \rho_0$. The comparison between A18 and A8' stresses, once again, the anomalous behavior of the latter potential, being excessively attractive at large densities with respect to the complete A18 model. Therefore, we stress once again that one should bear in mind this fact if A8' is going to be used to study massive and dense objects, as neutron stars.

In this paper we have compared the energies of homogeneous matter of nucleons (symmetric nuclear matter and pure neutron matter) as obtained by means of several modern many-body theories. We have used realistic potentials with and without spin-orbit components, in order to assess their importance and the accuracy of their evaluation. The methods employed here include the Correlated Basis Functions and the Bethe-Brueckner-Goldstone theories, at different levels of implementation. We have also presented and discussed the quantum Monte Carlo results available for neutron matter. For neutron matter and in absence of spin-orbit interaction all methods, but perturbative CBF, are in good agreement up to $\rho \sim 3 \rho_0$. After the inclusion of the LS components the agreement still persists around ρ_0 ,

whereas it is spoiled at increasing density. In particular, the accuracy of the CBF approach seems to become questionable because of the relevance of the missing terms. All methods provide a contribution of the spin-orbit potential $\Delta E_{LS} \sim -3 - 4$ MeV per nucleon. The AFDMC and the constrained path GFMC methods only give a lower contribution $\Delta E_{LS} \sim -0.1 - 1.2$ MeV per nucleon. In symmetric nuclear matter, where no Monte Carlo results are available, BHF and variational theories are in agreement up to $\sim 1.5 \rho_0$. Beyond this density, we still find good agreement for the potential having LS components, and less without. Quantum Monte Carlo results for SNM are much needed to clarify the behavior of the equation of state at large densities.

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- [1] V.R. Pandharipande, I. Sick, and P. deWitt Huberts, Rev. Mod. Phys., **69**, 981 (1997).
 - [2] S. Fantoni, and V.R. Pandharipande, Nucl. Phys. **A427**, 473 (1984).
 - [3] P. Grangè, J. Cugnon, and A. Lejeune, Nucl. Phys. **A473**, 365 (1987).
 - [4] A. Ramos, A. Polls, and W.H. Dickoff, Nucl. Phys. **A503**, 1 (1989).
 - [5] M. Baldo, I. Bombaci, G. Giansiracusa, U. Lombardo, C. Mahaux, and R. Sartor, Phys. Rev. **C41**, 1748 (1990).
 - [6] O. Benhar, A. Fabrocini, and S. Fantoni, Phys. Rev. **C41**, R24 (1990).
 - [7] M. Baldo, I. Bombaci, G. Giansiracusa, and U. Lombardo, Nucl. Phys. **A530**, 135 (1991).
 - [8] B.-A. Li, Phys. Rev. Lett. **88**, 192701 (2002); Phys. Rev. **C69**, 034614 (2004).
 - [9] I. Bombaci, in *Isospin Physics in Heavy-Ion Collisions at Intermediate Energies*. Edited by B.-A. Li and W.U. Schröder, (Nova Science Publisher, NewYork, 2001), p. 35.
 - [10] R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).

- [11] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, and J.J. de Swart, Phys. Rev. C **49**, 2950 (1994).
- [12] R. Machleidt, F. Sammarruca, and Y. Song, Phys. Rev. C **53**, R1483 (1996).
- [13] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, and J.J. de Swart, Phys. Rev. C **48**, 792 (1993).
- [14] R.B. Wiringa, S.C. Pieper, J. Carlson, and V.R. Pandharipande, Phys. Rev. C **62**, 014001 (2000).
- [15] S.C. Pieper, K. Varga, and R.B. Wiringa, Phys. Rev. C **66**, 044310 (2002).
- [16] A. Fabrocini, and S. Fantoni, in *First Course on Condensed Matter*, ACIF series, vol.VIII, D. Prosperi, S. Rosati, and G. Violini eds., World Scientific, Singapore (1986)87.
- [17] S. Fantoni and A. Fabrocini, in *Microscopic Quantum Many-Body Theories and their Applications* edited by J. Navarro and A. Polls, Lecture Notes in Physics, Vol. 510 (Springer-Verlag, Berlin 1998).
- [18] M. Baldo, in *Nuclear Methods and the Nuclear Equation of State*, M. Baldo editor, World Scientific, Singapore (1999).
- [19] V.R. Pandharipande, and R.B. Wiringa, Rev. Mod. Phys. **51**, 821 (1979).
- [20] H.Q. Song, M. Baldo, G. Giansiracusa, and U. Lombardo, Phys. Rev. Lett. **81**, 1584 (1998).
- [21] K.E. Schmidt, and S. Fantoni, Phys. Lett. B **446**, 99 (1999).
- [22] K.E. Schmidt and M.H. Kalos, in *Monte Carlo Methods in Statistical Physics*, edited by K. Binder (Springer-Verlag, Berlin, 1984).
- [23] S. Fantoni, A. Sarsa, and K.E. Schmidt, Phys. Rev. Lett. **87**, 181101 (2001).
- [24] J. Carlson, J. Morales, V.R. Pandharipande, and D.G. Ravenhall, Phys. Rev. C **68**, 025802 (2003).
- [25] M. Viviani, private communication.
- [26] B.S. Pudliner, V.R. Pandharipande, J. Carlson, S.C. Pieper, and R.B. Wiringa, Phys. Rev. C **56**, 1720 (1997).
- [27] R.B. Wiringa, V. Ficks, and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988).
- [28] J. Morales, V.R. Pandharipande, and D.G. Ravenhall, Phys. Rev. C **66**, 054308 (2002).
- [29] A. Fabrocini, F. Arias de Saavedra, G. C3, and P. Folagarait, Phys. Rev. C **57**, 1668 (1998).
- [30] M. Baldo, A. Fiasconaro, H.Q. Song, G. Giansiracusa, and U. Lombardo, Phys. Rev. C **65**, 017303 (2001).

- [31] R. Rajaraman, and H. Bethe, Rev. Mod. Phys. **39**, 745 (1967).
- [32] X.R. Zhou, G.F. Burgio, U. Lombardo, H.-J. Schulze, and W. Zuo, Phys. Rev. C **69**, 018801 (2004).
- [33] M. Baldo, and C. Maieron, Phys. Rev. C **69**, 014301 (2004).
- [34] A. Sarsa, S. Fantoni, K.E. Schmidt, and F. Pederiva, Phys. Rev. C **68**, 024308 (2003).
- [35] L. Brualla, S. Fantoni, A. Sarsa, K.E. Schmidt, and S.A. Vitiello, Phys. Rev. C **67**, 065806 (2003).
- [36] H. Mütter, and A. Polls, Phys. Rev. C **61**, 014304 (1999).

	PNM						SNM		
ρ/ρ_0	E_0^v	E_0^{CBF}	E_0^{BHF}	E_0^{BBG}	E_0^{GFMC}	E_0^{AFDMC}	E_0^v	E_0^{CBF}	E_0^{BHF}
0.5	10.10	9.72	9.79	9.67	9.54(03)		-7.54	-9.88	-9.91
1.0	15.38	13.61	15.89	15.09	14.81(11)	15.12(4)	-12.23	-14.09	-13.77
1.5	21.17	18.20	23.02	21.26	20.65(08)		-15.18	-17.49	-14.50
2.0	27.96	23.91	31.32	28.74		27.84(6)	-16.47	-19.43	-13.66
2.5	35.84	30.70	40.83	37.55		36.00(10)	-16.26	-19.62	-11.48
3.0	44.76	38.60	51.55				-14.65	-19.07	-8.16

TABLE I: Energies per nucleon, in MeV, for PNM and SNM in different approaches with the A6' potential. GFMC and AFDMC statistical errors in parenthesis.

	PNM						SNM		
ρ/ρ_0	E_0^v	E_0^{CBF}	E_0^{BHF}	E_0^{BBG}	E_0^{GFMC}	E_0^{AFDMC}	E_0^v	E_0^{CBF}	E_0^{BHF}
0.5	8.74	8.46	8.28	8.25	8.40(01,-1.1)		-8.99	-11.93	-12.02
1.0	11.53	9.56	11.99	11.83	11.90(27,-5.1) 14.98(6)		-15.32	-17.86	-18.20
1.5	13.93	10.17	16.03	15.85	16.85(50,-11.5)		-20.26	-24.06	-22.19
2.0	16.64	10.93	20.93	21.95		27.3(1)	-23.87	-28.85	-24.67
2.5	19.90	12.80	26.66	29.04		35.3(1)	-26.22	-32.31	-26.03
3.0	23.86	15.36	33.23				-27.30	-34.69	-26.50

TABLE II: Energies per nucleon, in MeV, for PNM and SNM in different approaches with the A8' potential. GFMC statistical errors (first number) and box corrections (second number) in parenthesis. AFDMC statistical errors in parenthesis.

	PNM/A8'		PNM/A6'		SNM/A8'		SNM/A6'	
$\rho/\rho_0 \rightarrow$	1.0	2.0	1.0	2.0	1.0	2.0	1.0	2.0
$\langle v_c \rangle$	-6.43	-11.08	-7.80	-13.84	-0.63	-2.62	0.60	-5.02
$\langle v_\tau \rangle$	1.20	2.79	1.05	2.58	-1.70	-2.36	-3.12	-2.59
$\langle v_\sigma \rangle$	-8.32	-11.85	-8.73	-12.67	-3.83	-5.34	-4.52	-6.67
$\langle v_{\sigma\tau} \rangle$	-10.39	-14.44	-10.92	-15.45	-20.62	-28.00	-23.92	-29.10
$\langle v_t \rangle$	0.10	-0.14	0.10	-0.01	0.27	0.40	0.12	0.26
$\langle v_{t\tau} \rangle$	-3.18	-6.20	-2.69	-5.65	-32.07	-46.03	-25.70	-39.01
$\langle v_{LS} \rangle$	-6.14	-15.82			-0.77	-7.14		
$\langle v_{LS\tau} \rangle$	-2.85	-7.41			-8.13	-14.76		

TABLE III: Contributions to the BHF energy per nucleon, in MeV, from the various components of the A6' and A8' potentials.

	A18				A8'	
$k_F[\text{fm}^{-1}]$	BHF	BHF/BF	BBG	FHNC	BHF	ρ/ρ_0
1.0	-10.84	-11.00	-7.67	-7.61	-11.72	0.42
1.2	-13.62	-14.13	-12.83		-15.16	0.73
1.4	-16.00	-16.43	-16.22	-14.43	-19.69	1.16
1.6	-16.12	-16.30	-17.00		-23.49	1.73
1.8	-11.88	-11.31	-13.24	-16.51	-25.94	2.46
2.0	0.25	1.14	1.04		-26.41	3.38
2.2	23.17	24.49	28.03	-6.88	-24.85	4.50

TABLE IV: Comparison between the BHF, the BHF/BF and BBG of Ref.[30] and FHNC/SOC SNM energies per nucleon (in MeV) for the A18. The BHF results for A8' are also reported.